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# The cosmological constant and entropy problems: mysteries of the present with profound roots in the past

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## Abstract

An accelerated universe should naturally have a vacuum energy density determined by its dynamical curvature. The cosmological constant is most likely a temporary description of a dynamical variable that has been drastically evolving from the early inflationary era to the present. In this Essay we propose a unified picture of the cosmic history implementing such an idea, in which the cosmological constant problem is fixed at early times. All the main stages, from inflation and its (“graceful”) exit into a standard radiation regime, as well as the matter and dark energy epochs, are accounted for. Finally, we show that for a generic Grand Unified Theory associated to the inflationary phase, the amount of entropy generated from primeval vacuum decay can explain the huge measured value today.

Key words: cosmology: dark energy, cosmology: theory

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## I. THE COSMOLOGICAL CONSTANT PROBLEM

One of the most perplexing aspects of the cosmological constant (CC) or  $\Lambda$ -problem [1–4] is that the current energy density  $\rho_{\Lambda 0} = \Lambda/(8\pi G)$  of vacuum is of order  $10^{-47} \text{ GeV}^4 \sim (10^{-3} \text{ eV})^4$  in natural units ( $G$  is Newton’s constant). Only a light neutrino of a millielectronvolt could be associated to such low energy density. Any other known particle provides a contribution which is exceedingly much bigger, e.g.  $m_e^4/\rho_{\Lambda 0} \sim 10^{34}$  for the electron.

With the advent of the Higgs boson discovery, the existence of the electroweak vacuum is generally considered a proven fact. Denoting by  $M_H \simeq 125 \text{ GeV}$  the (measured) Higgs mass and  $M_F \equiv G_F^{-1/2} \simeq 293 \text{ GeV}$  the Fermi scale, the zero-point energy (ZPE) from the Higgs field is of order  $M_H^4 \sim 10^8 \text{ GeV}^4$ , and the ground state value of the (classical) Higgs potential reads  $\langle V \rangle = -(1/8\sqrt{2})M_H^2 M_F^2 \sim -10^9 \text{ GeV}^4$ . In magnitude this is of order  $\sim v^4$ , where  $v \sim 250 \text{ GeV}$  is the vacuum expectation value of the scalar field. Equally significant is the ZPE from the top quark (with mass  $m_t \simeq 174 \text{ GeV}$ ), which is of order  $m_t^4 \sim 10^9 \text{ GeV}^4$  and negative (because it is a fermion). After adding up all these effects a result of the same order ensues which is  $(10^9/10^{-47}) \sim 10^{56}$  times bigger than what is needed. Even if by some miracle they would conspire to give zero at the tree-level, higher order effects (both from the ZPE and the Higgs potential) would spoil the cancelation until  $\sim 20$ th order of perturbation theory – still rendering a quantum payoff of order  $\rho_{\Lambda 0}$ :

$$\delta\rho_\Lambda \sim \left( \frac{g^2}{16\pi^2} \right)^{20} M^4 \sim 10^{-47} \text{ GeV}^4. \quad (1)$$

Here  $g$  stands typically for the  $SU(2)$  electroweak gauge coupling, and  $M$  is an effective mass of order  $\sim v$ . Accordingly, the very many thousand electroweak loops contributing all the way down from 20th order of perturbation theory to zeroth order should be carefully, and somehow magically, adjusted. Truly bewildering, if formulated on these grounds!

## II. RENORMALIZING AWAY $\Lambda$

What could possibly be wrong in the above argument? Most likely this: to accept uncritically that the  $\sim m^4$  contribution from a particle of mass  $m$ , and the  $\sim v^4$  one from the Higgs potential, are both individual *physical* contributions to  $\rho_\Lambda$ . Let us have a closer look at the origin of the  $\sim m^4$  effects by considering a real scalar particle in the context of

quantum field theory in curved spacetime [5]. The one-loop correction to the effective action can be computed e.g. in the  $\overline{\text{MS}}$  scheme in  $N$  dimensions. After setting  $N \rightarrow 4$ , except in the poles, one arrives at:

$$\Gamma_{\text{eff}}^{(1)} = \frac{1}{32\pi^2} \int d^4x \sqrt{-g} \left( \frac{2}{4-N} + \ln \frac{\mu^2}{m^2} + C \right) \left( \frac{1}{2} m^4 - m^2 a_1(x) + a_2(x) + \dots \right), \quad (2)$$

where  $C$  is a constant. The  $a_{1,2}$  are the so-called Schwinger-DeWitt coefficients (coming from the adiabatic expansion of the matter field propagator in the curved background). They have a purely geometric form given by a linear combination of the curvature  $R$  and the higher derivative (HD) terms  $R^2$ ,  $R_{\mu\nu}^2$  etc. If the starting classical action contains already the usual Einstein-Hilbert (EH) term and the HD terms, all divergences can be absorbed by the counterterms generated from the bare parameters  $\rho_\Lambda^{(b)}$ , inverse Newton's coupling and the coefficients  $\alpha_i^{(b)}$  of the various higher order  $\sim R^2$ :

$$\rho_\Lambda^{(b)} = \rho_\Lambda(\mu) + \delta\rho_\Lambda, \quad \frac{1}{G^{(b)}} = \frac{1}{G(\mu)} + \delta \left( \frac{1}{G} \right), \quad \alpha_i^{(b)} = \alpha_i(\mu) + \delta\alpha_i. \quad (3)$$

As usual, each bare quantity splits into the renormalized part (carrying an arbitrary scale  $\mu$ ) and a counterterm, which is then chosen to cancel the corresponding divergence (pole at  $N = 4$ ). Gathering the various pieces that go into the renormalization of the vacuum energy density, we are led to the expression:

$$\rho_\Lambda(\mu) + \delta\rho_\Lambda - \frac{m^4}{64\pi^2} \left( \frac{2}{4-N} + \ln \frac{\mu^2}{m^2} + C \right). \quad (4)$$

In the  $\overline{\text{MS}}$  scheme the counterterm reads  $\delta\rho_\Lambda^{\overline{\text{MS}}} = (m^4/64\pi^2) \left( \frac{2}{4-N} + \text{const.} \right)$  and the one-loop renormalized result takes on the form:

$$\rho_{\text{vac}}^{(1)} = \rho_\Lambda(\mu) + \frac{m^4}{64\pi^2} \left( \ln \frac{m^2}{\mu^2} + C_{\text{vac}} \right), \quad (5)$$

where  $C_{\text{vac}}$  is a finite constant. We remark that the very same result (5) is obtained from the (much) simpler calculation of the ZPE in flat spacetime, starting e.g. from the dimensionally regularized sum of half frequencies  $\sum_k \frac{1}{2} \omega_k$  in the continuum limit. The difference is that, in the curved spacetime treatment (2), the geometric terms also appear and involve corrections to the EH action (hence to Newton's coupling) and the HD terms  $\sim R^2$ .

From Eq. (5) one usually concludes that a particle of mass  $m$  contributes a quantum correction  $\sim m^4$  to the value of the CC; this is what triggers the preposterous fine tuning problem outlined in the beginning. However, we can take another viewpoint/ansatz.

Recall that the counterterm depends on an arbitrary constant and that the renormalized  $\rho_\Lambda(\mu)$  is, albeit finite, *not* a physical quantity. Thus, being the formal expression (5) the same in flat spacetime, a more natural renormalization condition is to arrange for the exact cancelation of both the UV *and* the finite parts with the counterterm. In other words, we should set the expression (4) exactly to zero. In this way  $\rho_{\text{vac}}^{(1)} = 0$  in flat spacetime rather than the result (5) – incompatible with Einstein’s equations in that background. By the same token the Higgs yield  $\sim v^4$ , which is part of the flat space result, is included in that prescription. Overall the CC is zero at this point and flat spacetime can be a solution of the field equations in the absence of expansion. There is no fine tuning now: for we did not adjust finite numbers; rather, we renormalized an infinite quantity carrying an arbitrary finite part, similar to what is done e.g. with the mass and charge of the electron in QED. Furthermore, the renormalization is meaningful as it is carried out in the UV regime, where we have the reliable tools of QFT to effectively implement the renormalization program.

### III. RUNNING VACUUM

Removing the flat spacetime result (5) is similar to subtracting the free space part in the Casimir effect so as to project the vacuum vibrational modes in between the plates only. In our case what remains, after renormalizing away the unwanted terms in the CC, are just the EH and HD curvature effects. But we expect something else, to wit: the genuine vacuum contributions related to the expanding background. They should also be purely geometric and of quantum nature, for only in this way can they be as small as are currently observed. The leading effects may generically be captured from a renormalization group equation of the form

$$\frac{d\rho_\Lambda}{d\ln H^2} = \frac{1}{(4\pi)^2} \sum_i \left[ a_i M_i^2 H^2 + b_i H^4 + c_i \frac{H^6}{M_i^2} + \dots \right]. \quad (6)$$

This equation describes the rate of change of  $\rho_\Lambda$  with the Hubble function  $H(t)$ , acting here as the running scale in the FLRW metric – see [6] for a review and a comprehensive list of references. The r.h.s. of (6) represents the  $\beta$ -function of  $\rho_\Lambda$ . It can involve *only* even powers of the Hubble rate  $H$  (because of the covariance of the effective action) [7, 8]. The coefficients  $a_i, b_i, c_i, \dots$  are dimensionless, and the  $M_i$  are the masses of the particles in the loops. For a concrete scenario of this kind within anomaly-induced inflation, see [8]; and for

an implementation within dynamically broken Supergravity cf. [9]. Related developments within the renormalization group approach in cosmology are presented in [10].

Integrating the above equation and keeping, for the sake of illustration, only one of the higher powers  $H^{n+2}$ , we can express the result as follows:

$$\rho_\Lambda(H) = \frac{3M_P^2}{8\pi} \left[ c_0 + \nu H^2 + \frac{H^{n+2}}{H_I^n} \right] \quad (n \geq 2), \quad (7)$$

with  $c_0$  an integration constant and  $H_I$  a parameter, both dimensionful [11]. For a generic GUT, the dimensionless parameter  $\nu = \frac{1}{6\pi} \sum_{i=f,b} a_i \frac{M_i^2}{M_P^2}$  is feeded by the heavy masses  $M_i$  of boson and fermions relative to the Planck mass  $M_P$ . Typically  $|\nu| = 10^{-6} - 10^{-3}$  [8].

#### IV. INFLATION AND GRACEFUL EXIT

We adopt (7) as a prototype for a “unified running vacuum model” in the framework of the gravitational field equations within the FLRW metric in flat 3-dimensional space:

$$3H^2 = 8\pi G(\rho_m + \rho_\Lambda(H)), \quad 2\dot{H} + 3H^2 = -8\pi G(\omega_m \rho_m - \rho_\Lambda(H)). \quad (8)$$

Taking  $\omega_m = 1/3$  for the equation of state parameter of the relativistic matter fluid in the early universe, and neglecting  $\nu$  and  $c_0/H^2$  at this stage, we can solve for  $H$  and the energy densities in terms of the scale factor. We find:

$$H(\hat{a}) = \frac{H_I}{(1 + \hat{a}^{2n})^{1/n}} \quad (9)$$

along with

$$\rho_\Lambda(\hat{a}) = \frac{\rho_I}{f(\hat{a})}, \quad \rho_r(\hat{a}) = \frac{\rho_I \hat{a}^{2n}}{f(\hat{a})}, \quad (10)$$

where

$$f(\hat{a}) \equiv (1 + \hat{a}^{2n})^{1+2/n}. \quad (11)$$

Here  $\hat{a} = a/a_{\text{eq}}$  is the scale factor, normalized to the value  $a_{\text{eq}}$  where the decaying vacuum density equals the radiation density (i.e.  $\rho_\Lambda = \rho_r$  at  $\hat{a} = 1$ ); and  $\rho_I = 3H_I^2/8\pi G$  is the (finite) critical density at  $a = 0$ .

At the initial point of the cosmic evolution,  $\rho_\Lambda(0) = \rho_I$  and  $\rho_r(0) = 0$ , so the model is nonsingular. Furthermore, for a typical GUT scale  $M_X \sim 10^{16}$  GeV associated to the inflationary epoch, we find  $H(\hat{a}) < H_I \sim \sqrt{\rho_I}/M_P \sim (M_X/M_P)^2 M_P < 10^{-5} M_P$  since

$\rho_I \sim M_X^4$ . This result is in agreement with the well known CMB bound on the fluctuations induced by the tensor modes [12].

Let us briefly highlight the expansion and thermal histories. In the beginning we have  $\hat{a} \rightarrow 0$  and  $f(\hat{a}) \rightarrow 1$ , and hence  $\rho_\Lambda \simeq \rho_I = \text{const.}$  and  $\rho_r \rightarrow 0$ . Thus the universe starts with no matter at all; it contains just vacuum energy and as a result grows exponentially fast:  $a(t) \propto e^{H_I t}$ . At the same time  $\rho_r(\hat{a})$ , which starts from zero, increases very fast as  $\sim \hat{a}^{2n}$  at the expense of vacuum decay. Much later (when  $\hat{a} \gg 1$ ,  $f(\hat{a}) \rightarrow \hat{a}^{2n+4}$ ) we attain the asymptotic regime within the radiation epoch, in which the relativistic matter density decays as  $\rho_r \sim a^{-4}$  ( $a \sim t^{1/2}$ ). Thus we achieve “graceful exit” from inflation into the standard radiation epoch, a remarkable feature which – we should emphasize – holds good for *any*  $n$  in (7).

## V. SOLVING THE COSMOLOGICAL ENTROPY PROBLEM

The temperature of the heat bath generated from primeval vacuum decay follows from equating  $\rho_r(a)$  to the black-body form  $(\pi^2/30)g_*T_r^4$ , where  $g_*$  is the number of active d.o.f. Thus  $T_r(\hat{a}) = T_X \hat{a}^{n/2}/f^{1/4}(\hat{a})$ , where  $T_X \equiv (30\rho_I/\pi^2 g_*)^{1/4} \sim M_X$  is of the order of the maximum attained temperature. The radiation entropy  $S_r = (4\rho_r/3T_r) a^3$  [13] now yields:

$$S_r(\hat{a}) = \left( \frac{4\rho_I}{3T_X} \right) g(\hat{a}) a_{\text{eq}}^3, \quad g(\hat{a}) \equiv \frac{\hat{a}^{3(1+n/2)}}{[1 + \hat{a}^{2n}]^{\frac{3}{4}(1+2/n)}}. \quad (12)$$

It rises extremely fast in the beginning:  $S \sim \hat{a}^{3(1+n/2)}$  (e.g  $S \sim \hat{a}^6$  for  $n = 2$ ). But deep in the radiation epoch  $\hat{a} \gg 1$  (i.e. for  $a \rightarrow a_r \gg a_{\text{eq}}$ ) we have  $g(\hat{a}) \rightarrow 1$ , and  $S_r$  rapidly stagnates at the asymptotic value  $S_r \rightarrow S_\infty \equiv (4\rho_I/3T_X) a_{\text{eq}}^3 = (2\pi^2/45)g_*T_X^3 a_{\text{eq}}^3$ . Upon inspecting once more the temperature  $T_r(\hat{a})$ , we find that in the beginning it also rises fast:  $T_r \sim \hat{a}^{n/2}$ , but for  $a \rightarrow a_r$  it eventually adapts to the adiabatic behavior  $T_r = T_X/\hat{a}_r$  (for *any*  $n$ ), i.e.

$$T_X a_{\text{eq}} = T_r a_r, \quad (\hat{a} \gg 1), \quad (13)$$

thereby the asymptotic entropy can be cast as

$$S_\infty = (2\pi^2/45)g_*T_r^3 a_r^3. \quad (14)$$

It is precisely during this adiabatic phase when the quantity  $g_*T_r^3 a_r^3$  becomes conserved and equals the current value  $g_{s,0}T_{\gamma 0}^3 a_0^3$ , in which  $T_{\gamma 0} \simeq 2.725^\circ\text{K}$  (CMB temperature now)

and  $g_{s,0} = 2 + 6 \times (7/8) (T_{\nu,0}/T_{\gamma 0})^3 \simeq 3.91$  is the entropy factor for the light d.o.f. today, computed from the ratio of the present neutrino and photon temperatures. The upshot is that the entropy enclosed in our horizon today,  $H_0^{-1}$ , namely

$$S_0 = \frac{2\pi^2}{45} g_{s,0} T_{\gamma 0}^3 (H_0^{-1})^3 \simeq 2.3 h^{-3} 10^{87} \sim 10^{88} \quad (h \simeq 0.67), \quad (15)$$

can be fully accounted for from the asymptotic value  $S_\infty$  in the radiation epoch. Such number, therefore, was deeply encoded in our remote past and – remarkably enough – does not depend neither on the details of the GUT nor on the value of  $n$  in Eq. (7). This result generalizes the one found for  $n = 2$  in [14] within the context of an arbitrary GUT, and also the result of [15] in the different context of assuming a Gibbons-Hawking initial temperature. This finding, in its various versions, might provide a new solution to the entropy/horizon problems [13].

## VI. THE CURRENT UNIVERSE

Finally, in the matter-dominated epoch:  $H^{2+n} \ll H^2$  in (7). The field equations can now be solved anew (with  $\omega_m = 0$ ) using the neglected terms during the de Sitter period. The energy densities are easily found:

$$\rho_m(a) = \rho_{m0} a^{-3(1-\nu)}, \quad \rho_\Lambda(a) = \rho_{\Lambda 0} + \frac{\nu \rho_{m0}}{1-\nu} [a^{-3(1-\nu)} - 1], \quad (16)$$

with  $8\pi G\rho_{\Lambda 0} = 3c_0 + 3\nu H_0^2$ . They clearly follow the  $\Lambda$ CDM behavior provided the condition  $|\nu| \ll 1$  holds [16, 17], as theoretically expected [8]. Hints of dynamical vacuum energy can indeed be detected even for  $|\nu|$  as small as  $10^{-3}$ , see Ref.[18].

It is also remarkable that the small transfer of energy between vacuum and matter embodied in (16), which is parameterized by  $|\nu| \ll 1$ , can be interpreted (see [19–21] and references therein) as a time variation of the particle masses (both from baryons and dark matter) and the fundamental “constants” of Nature. It is therefore tempting to propose (according to the aforementioned references) that such dynamical vacuum framework may offer a possible explanation within General Relativity for the numerous hints suggesting a small cosmic drift of their values – see the recent [22] and references therein. See also Ref. [23] for a summarized introduction to this fascinating subject.

## VII. CONCLUSIONS

In summary, the cosmological constant problem seems to be connected with the terms that also appear when one naively computes the vacuum energy in flat spacetime, whereas the (much smaller) contributions related with the dynamical curvature of the expanding Universe can be described in an effective way by quantum effects that follow a renormalization group equation driven by the Hubble flow. As a result the Dark Energy that we measure today from the accelerated phase of our Cosmos should ultimately be the effect of the dynamical vacuum energy of the expanding background. After renormalizing away the terms that are in common with flat spacetime, we are left with a vacuum energy density and entropy which nicely resonate with all the main traits of the cosmic history, and with a dynamical tail that may be the “smoking gun” of this mechanism. It all happens as though some of the most puzzling mysteries of our present may have indeed profound roots in the distant past – those early times when our Universe encoded the fundamental seeds of its entire future evolution.

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